Stringy gravity, interacting tensionless strings and massless higher spins

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Consequences of a strong version of the AdS/CFT correspondence for extremely stringy physics are examined. In particular, properties of $\mathcal{N}=4$ supersymmetric Yang-Mills theory are used to extract results about interacting tensionless strings and massless higher spin fields in an $AdS_5 \times S^5$ background. Furthermore, the thermodynamics of this model signals the presence of a Hawking-Page phase transition between AdS_5 space and a "black hole"-like high temperature configuration even in the extreme string limit.

1. INTRODUCTION

The properties of string theory at extreme energies has been a deep puzzle for many years. In the spirit of the history of asymptotically free field theories we would expect an understanding of short-distance symmetries to be of fundamental importance for an efficient formulation of the theory and perhaps even for its consistency. Many important aspects of the problem have been illuminated, like symmetries of the S-matrix [1,2] and a stringy uncertainty principle [3,4], but progress has been hampered by a fundamental obstacle: String theory is finite rather than asymptotically free. Finiteness means that we typically have to worry about the full perturbation expansion, which diverges [5]. A non-perturbative formulation like field theory would then save the day, but unfortunately a tractable field theory formulation is lacking for closed strings.

A closed-string field theory pending, it is still true that there has been enormous progress in the understanding of non-perturbative string physics in recent years. The increased knowledge has however usually been based on dualities between low energy field theories, generalized to the full string theories. Only Maldacena's AdS/CFT correspondence [6–9] offers the hope of replacing an "untractable" string theory with a better known quantum field theory, but it could seem unnecessarily complicated from a high-energy perspective, because the string theory lives in a curved AdS space.

It has been argued [10] that a tensionless string theory should constitute the proper starting point for investigating short-distance string theory, just like massless field theories are high energy limits of massive theories. Similarly, broken gauge symmetries for non-zero tension could be expected to be restored in the tensionless limit. Quantization of tensionless strings in flat spacetime does however prove to be problematic. The method used in [10] does not lead to a unique diagnosis, but one possible explanation of the problems could be that the flat background which was used is in fact inconsistent. Evidence for this interpretation comes from the study of theories of massless higher spin fields [11–14]. Such fields could be expected to appear in the tensionless limit of string theory, since there are fields of arbitrarily high spin in string theory and the masses of the free theory are all proportional to the string tension. Remarkably, Fradkin and Vasiliev [12,13] have observed that consistent interactions of the higher spin fields require a cosmological constant! In fact, the theories make sense precisely in AdS backgrounds. Thus Maldacena's AdS/CFT conjecture actually seems to be a very promising tool for studying the short-distance behaviour of string theory.

The present talk addresses how the correspon-

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dence can be applied to the study of short-distance properties of string theory. It is mainly based on [15,16], which also contain more complete references, but the emphasis is different and several new observations are made.

1.1. AdS/CFT

The prime example of the AdS/CFT correspondence equates type IIB string theory on $AdS_5 \times S^5$ with a constant five-form Ramond-Ramond field strength to $\mathcal{N} = 4$ supersymmetric Yang-Mills theory [6]. The rank of the Yang-Mills gauge group $U(N)$ is directly related to the quantized RR flux N. For large N the Yang-Mills perturbation expansion is conveniently organised in a topological expansion [17] where successive terms appear with powers of $1/N$, and the effective loop expansion parameter within each topology is $g_{YM}^2 N$ rather than g_{YM}^2. For our purposes it is also important to keep track of the length scales of the string theory. Modulo numerical factors the correspondence gives $g_{YM}^2 N = R^4 T^2$, where T is the string tension and R is the radius of curvature of AdS_5 and S^5. Thus zero string tension corresponds to a free gauge theory! The remaining expansion parameter is somewhat more puzzling, but can be identified with the ratio of the Planck length to the radius of curvature, $1/N = \sqrt{G_5/R^5}$. Note that in contrast to what one might have expected from the conventional association of string world-sheets of different genera to the topological expansion, $1/N$ is *not* the ordinary string coupling (which vanishes) but a rescaled version. Clearly, finite N SYM provides a natural resummation of the genus expansion!

Here it must be pointed out that although there is an impressive amount of evidence for the Maldacena conjecture, the original argument and most of the evidence applies to *large* $g_{YM}^2 N$, and not to the opposite limit, which we consider here. Still, there is not yet any evidence against this strong form of the conjecture, and the discussion of thermodynamics I will present towards the end actually justifies a belief that the the dependence on $g_{YM}^2 N$ is smooth. Furthermore, we are dealing here with the most symmetric case of the correspondence, and many relations are protected by supersymmetry.

Nevertheless, the statement that tensionless string theory is just *free* SYM theory seems too simple. Indeed, we will find both actual amplitudes that can be very complicated, and a thermodynamic behaviour which exhibits a phase transition, so something more has to be hidden somewhere in the correspondence. The answer to the riddle of course lies in the way string physics is encoded in SYM theory. The recipes for relating generating functions [7,8], and for calculating amplitudes [18,19] involve SYM correlation functions of traces of products of fields, which generically give rise to very complicated combinatorics, even if we only have to evaluate free propagators. We have a theory of *interactions without interactions*. Dynamics intimately related to statistics and combinatorics is what we want in a fundamental theory of gravity: black holes as solutions to highly non-linear equations and black holes as thermodynamic objects should appear as different sides of the same coin.

I shall end this presentation with a description of how black hole thermodynamics can be reproduced, but first I want to discuss relevant aspects of $\mathcal{N} = 4$ SYM at weak coupling, how tensionless strings appear, and how massless higher spin theory is embedded in SYM theory.

2. $\mathcal{N} = 4$ SYM AT WEAK COUPLING

In $\mathcal{N} = 4$ SYM theory scalars, fermions and vectors are all in the adjoint representation of the gauge group. All local gauge-invariant observables are polynomials in traces of products of these adjoint fields and their derivatives. Single-trace operators thus constitute basic building blocks of all local observables. In the free theory their correlation functions may be used to compute any other correlation functions of local observables. Then different fundamental fields propagate independently so it is perfectly consistent to restrict the attention to a subset of them. For simplicity we only consider conformal operators built of the six scalar fields ϕ^I (transforming under the R-symmetry group $SO(6)$).

All single-trace operators built of scalars are characterized solely by their index structure and how derivatives are distributed. We may for ex-

Table 1
Spectrum of relevant and marginal primaries composed of scalars in terms of $SO(6)$ Young tableaux.

Primary/Dimension	$\Delta = 2$	$\Delta = 3$	$\Delta = 4$
Φ^{IJ}	$\ydiagram{2} \oplus \bullet$		
Φ^{IJK}		$\ydiagram{3} \oplus \ydiagram{1} \oplus \ydiagram{1,1}$	
Φ^{IJKL}			$\ydiagram{4} \oplus 2\,\ydiagram{2} \oplus 2\,\bullet \oplus 2\,\ydiagram{2,2} \oplus 2\,\ydiagram{2,1} \oplus \ydiagram{1,1}$
$(\partial^{10}\Phi^{IJ})_\mu$		$\ydiagram{1,1}$	
$(\partial^{100}\Phi^{IJK})_\mu$			$2\,\ydiagram{2,1} \oplus 2\,\ydiagram{1}$
$(\partial^{\{\Sigma n=2\}}\Phi^{IJ})_{\mu\nu}$			$\ydiagram{2} \oplus \bullet$

The primaries in the last row have derivatives arranged so that they transform as symmetric traceless Lorentz tensor, and are thus generalized stress tensors.

ample write

$$\Phi^{IJ} \equiv \mathrm{Tr}\left\{\phi^I \phi^J\right\} \ , \tag{1}$$

and note that the antisymmetric part of this operator vanishes by the properties of the trace. Derivatives inside the traces are very significant unless they combine to equations of motion for the fundamental fields. In that case correlation functions vanish. Furthermore, overall derivatives on the operators produce new operators, descendants in CFT parlance, whose properties can be deduced immediately from the original operators, since they are essentially the results of infinitesimal translations. For instance, the symmetric part of

$$\left(\partial^{10}\Phi^{IJ}\right)_\mu \equiv \mathrm{Tr}\left\{\partial_\mu \phi^I \phi^J\right\} \ , \tag{2}$$

is a descendant, again by the properties of the trace. Here we have introduced a notation in which the symbolic exponent of the the derivative counts how many derivatives should be applied to the different fundamental factors in the trace. The non-trivial operator content of the theory is given by the primaries, which by definition cannot be written as descendants. The primaries can be classified in representations of the Lorentz algebra and $SO(6)$. The simplest operators are given in Table 1.

This table contains relevant (dimension $\Delta = 2,\ 3$) and marginal ($\Delta = 4$) primaries which are important because they may be added to the lagrangian in order to deform the theory in the IR, without ruining the UV behaviour. The Maldacena conjecture gives the dimensions of all operators in the strong coupling limit from IIB supergravity in $AdS_5 \times S^5$ [8]. They agree with the dimensions of so-called chiral primaries, which are independent of the coupling both classically and quantum-mechanically due to non-renormalization theorems. In our table totally symmetric scalar primaries belong to this class. The presence of other operators indicates that their dimensions depend strongly on the coupling and diverge at strong coupling. By continuity, operators which are relevant at zero coupling have to become marginal for some definite intermediate value of $g_{YM}^2 N = R^4 T^2$, implying the possibility of critical values of the string tension where marginal deformations can give rise to entirely new conformal field theories and to new realizations of the AdS/CFT correspondence.

3. SYM AND TENSIONLESS STRINGS

The single-trace operators admit a natural interpretation as strings composed of string bits constituted by the fundamental fields and their derivatives. (I use the term string bits here because it seems to capture the situation well, but there are both differences and similarities to Thorn's use of the term [20].) The operators propagate to other operators, with dominant contributions for large N corresponding to the free propagation of the individual string-bits to other strings preserving the same cyclic order[2] of the constituent fields, e.g.

$$\frac{1}{N^3} <: \left[\mathrm{Tr}(\phi^1\phi^2\phi^3)\right](x) :: \left[\mathrm{Tr}(\phi^3\phi^2\phi^1)\right](0) :> = |x|^{-6} \ , \quad (3)$$

$$\frac{1}{N^3} <: \left[\mathrm{Tr}(\phi^1\phi^2\phi^3)\right](x) :: \left[\mathrm{Tr}(\phi^1\phi^2\phi^3)\right](0) :> = \frac{1}{N^2}|x|^{-6} \ . \quad (4)$$

This free propagation of string bits is analogous to the free massless propagation of points on tensionless strings [10], although the present discussion only indirectly deals with propagation in the five-dimensional curved AdS space. At any rate, the shadow of string propagation on the four-dimensional CFT world behaves as expected of a tensionless string.

The single trace operators work combinatorically like holes cut out of world-sheets in the $1/N$ expansion[17], and thus correspond naturally to closed strings in correlation functions. The cyclic property of the trace also works as a discrete remnant of the reparametrization invariance of strings, and the effective vanishing of Green functions of such strings with string-bits proportional to equations of motion presumably corresponds to another remnant of the same symmetry.

General conformal field theory correlation functions are expected to be expressible in terms of pairwise operator product expansions

$$A(x)B(y) \sim \sum_D C_{AB}{}^D D(y)|x-y|^{\Delta_D-\Delta_A-\Delta_B} . \quad (5)$$

A four-point function of conformal operators can expanded in three different channels, and for higher point functions the number of channels increases. In the free case one can verify explicitly that the expressions in terms of diffferent channels agree. As pointed out in [16], given appropriate definitions of amplitudes in AdS backgrounds [18,19], this duality of conformal field theory translates to the channel duality of string amplitudes, which was originally the defining property of string theory [21]. In the $g^2_{YM}N = R^4T^2 = 0$ case CFT manifestly works as a string theory.

The AdS/CFT conjecture has another, very surprising consequence when applied to Table 1. Group theory of $SO(4,2)$ which is the conformal group of the CFT and the isometry group of AdS, dictates a relationship between mass and spin in AdS on one hand and dimension and spin in the CFT on the other hand. The bullet ($\bullet$) in the last row denotes an $SO(6)$ scalar traceless symmetric rank 2 tensor, the scalar contribution to the stress tensor, which is conserved. The stress tensor corresponds to the field with the same Lorentz transformation properties on the AdS side, i.e. the gravitational field. In addition, the conservation of energy-momentum leads to the reduction of the number of degrees of freedom appropriate for a massless field. This is all quite natural, but the argument is independent of $SO(6)$ representation and therefore also applies to the other operators in the last row of the table, which may easily be checked to be conserved. On the AdS side one appears to get several charged gravitational fields! This sounds truly strange, but only recently have arguments been given that can rule out such spectra [22]. This no-go theorem however relies both on locality and on a lagrangian formulation, assumptions which are not fulfilled in the present case.

4. MASSLESS HIGHER SPINS

Generalizations of the conserved $SO(6)$ and energy-momentum currents

$$\mathrm{Tr}\left\{\phi^I\partial_\mu\phi^J - \phi^J\partial_\mu\phi^I\right\} \ , \quad (6)$$

$$\mathrm{Tr}\left\{(\partial_\mu\phi^I\partial_\nu\phi^I - \frac{1}{2}\phi^I\partial_\mu\partial_\nu\phi^I) - \frac{\eta_{\mu\nu}}{4}\mathrm{trace}\right\} \ , \quad (7)$$

with more Lorentz indices, higher spin currents, have been studied in the literature [23–25,14]. For example, in the condensed notation of Vasiliev odd spin currents are

$$\mathrm{Tr}\left\{\partial_{\mu(n)}\phi^I\partial_{\mu(n+1)}\phi^J - (I \leftrightarrow J)\right\} \ , \quad (8)$$

where $\mu(n)$ denotes n symmetrized indices, and the symmetrization extends over all indices with

[2] The actual order in one of the traces in the Green function is reversed, since creation and annihilation operators of the same quantum are hermitean conjugates.

the same label (μ here). Even spin currents are instead symmetric in I and J. There are also unique improved versions of the currents which are traceless in the Lorentz indices [25], as appropriate in the present conformally symmetric case.

These currents are conserved for free fields, but not in other cases. Arguing in the same way as for the gravitational field we relate masslessness and gauge symmetries in AdS to conservation laws for dual CFT tensor currents. For the AdS/CFT correspondence applied to $\mathcal{N}=4$ SYM this means that the duals to higher spin currents should be massless at zero tension, but not for non-zero tension. In fact, to each conserved higher spin current corresponds precisely one massless higher spin gauge field in AdS [14]. That masslessness occurs at zero coupling of the CFT agrees well with expectations, since a string tension gives mass to all string states with higher spin than two.

To form high spin operators which are conserved and correspond to massless dual fields one needs as many derivatives as possible and few fundamental fields. Essentially, spin increases with number of derivatives, and for fixed number of derivatives mass increases with the numbers of fields. Since we have found massless higher spins for quadratic operators we expect all higher order operators to have massive duals. Due to tracelessness of the generators, linear operators are absent for the gauge group $SU(N)$ which has been argued to be the correct group in the AdS/CFT correspondence [8]. Then, the full set of higher spin currents are of the kind [23–25,14] described above.

It is interesting to compare results from the present approach based on AdS/CFT compared to the monumental earlier achievements, mainly by Vasiliev and co-workers. Among many other important facts these authors have established the structure of non-linear higher spin gauge symmetries and found interaction terms and equations of motion respecting the symmetries. The structure of the higher spin fields here is identical, including the symmetry pattern of symmetric IJ matrices at even spin and antisymmetric at odd spin. Thus the proliferation of graviton fields is independent of the string origin of our approach. In contrast, the massive fields seem to be special to the string theory approach. The main and very serious drawback is that we cannot write down a lagrangian or even the equations of motion for the higher spin fields. We can only calculate Green functions which may be interpreted as a kind of AdS scattering amplitude. On the other hand the full perturbation expansion in terms of the parameter $1/N$ may be summed by just calculating Green functions for a finite rank $SU(N)$ gauge group. Curiously, as we shall see more in detail, the spectrum of massive AdS fields depends on N in a complicated way. It would be interesting to know if the massive fields are more or less arbitrary additions to Vasiliev's higher spin theories, or if they serve some purpose for consistency, perhaps at the quantum level. If a strong connection between the approaches is established, the advanced mathematical machinery of higher spin gauge theories may perhaps be applied to solve problems in large N Yang-Mills theory.

5. 1/N COUNTING AND STRINGY BLACK HOLES

The spectrum of a theory is conveniently encoded in its thermodynamics, which also directly probes the physical behaviour of the theory. Thermodynamics plays an even more fundamental role in theories of gravitation, because of its mysterious relation to black holes. In the AdS/CFT correspondence these relations have been exploited by Witten [8,26] who observed that the Hawking-Page transition [27] in AdS gravity between thermal AdS space and an AdS black hole corresponds to the deconfinement transition in Yang-Mills theory. An essential ingredient is that the dependence on Newton's constant of the free energy of the black hole $\sim R^5/G_5$ is translated in gauge theory to N^2 by the AdS/CFT relation $1/N = \sqrt{G_5/R^5}$. Since the Einstein gravity argument by Hawking and Page is used, Witten's result strictly only applies to strongly coupled gauge theory. The zero coupling limit of the gauge theory which will be reviewed here was studied in [15] with the result that the phase transition persists! Translated back to AdS gravity, now in the guise of tensionless string theory or higher spin theory, some black hole like object survives even under these extreme conditions.

First some preliminaries. Witten's version of the AdS/CFT correspondence relates string theory on global AdS space to gauge theory on S^3. This makes sense if we postulate that the scalar fields are conformally coupled to the curvature. Then we can go between flat space and the sphere by a conformal mapping. Conformal dimensions in flat space are mapped to energies on S^3 (in units of the inverse radius of curvature). The problem of counting states of a definite energy amounts to counting gauge invariant local operators. The crucial step in the counting for $N \to \infty$ consists in counting single trace operators or tensionless strings. This problem may be solved by applying a combinatorial theorem due to Pólya, and it results in a Hagedorn-type [28] divergence of the free energy at a critical temperature of the order of the inverse radius. As one may suspect the behaviour above the Hagedorn temperature can be controlled only by taking N finite, but then other methods are needed.

In order to count all gauge singlets, single trace or not, with the appropriate statistics, the idea is to first form a partition sum keeping track of (global) $SU(N)$ rotations g in terms of the eigenvalues $R_i(g)$ of the representation matrices $R(g)$. Then the rotated partition sum for bosons

$$\begin{aligned}&\prod_k \prod_{i=1} (1 + x^{E_k} R_i(g) + x^{2E_k} R_i(g)^2 + \ldots)\\ &= \prod_k \mathrm{Det}(1 - x^{E_k} R(g))^{-1}\\ &= \exp\left(\sum_k \mathrm{Tr}\left(\sum_{n=1}^{\infty} \frac{x^{nE_k}}{n} R(g^n)\right)\right) \qquad (9)\end{aligned}$$

can be projected to singlet states by integration over the group, using the orthogonality properties of group characters $\chi(g)$. Taking also the fermion contribution into account yields

$$\int dg \exp\left(\sum_{n=1}^{\infty} \frac{\zeta_B(x^n) - (-1)^n \zeta_F(x^n)}{n} \chi(g^n)\right) \qquad (10)$$

where $\zeta_{B,F}(x^n)$ are partition sums of the free fundamental SYM boson and fermion fields (for $U(1)$ gauge group). $\chi(g)$ is the character of the adjoint representation

$$\chi(g) = N - 1 + 2 \sum_{m<n} \cos(\alpha_m - \alpha_n). \qquad (11)$$

The integrand of equation (10) then only depends on a set of N eigenvalues distributed over the unit circle, and not on their order. By a standard large N trick such integrals over many eigenvalues may be approximated by an integral over eigenvalue densities $\rho(\alpha)$. For large N we get a steepest descent estimate of the partition sum

$$Z(x) \propto \int D\rho \, e^{-S[\rho]}, \qquad (12)$$

by finding a ρ which makes S stationary. In our particular case one obtains an integral equation for ρ with qualitatively different solutions below and above the Hagedorn temperature. Below the transition the stationary eigenvalue density is constant over the whole range of eigenvalues, while above it develops “gaps”, i.e. the density gets concentrated on a subset of the full range which shrinks with increasing temperature. Above the transition one finds[3] for the free energy $F = -\log Z$ that

$$\begin{aligned}-\frac{2F}{N^2} &\approx \zeta(x) - 1 + \sqrt{\zeta(x)^2 - \zeta(x)}\\ &\quad - \log\left(\zeta(x) + \sqrt{\zeta(x)^2 - \zeta(x)}\right) \qquad (13)\end{aligned}$$

approximately, where $\zeta = \zeta_B + \zeta_F$. The essential point here is of course the N^2 factor and the sharp phase transition at the critical temperature which is similar to the Hawking-Page transition. Details on the behaviour close to the transition and on the interpretation as a Hagedorn transition may be found in [15].

6. CONCLUSIONS

I have tried to demonstrate in what sense free $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with gauge group $SU(N)$ behaves as an interacting theory of tensionless strings or equivalently as an interacting theory of massless higher spin fields in

[3] An almost identical calculation was done by Skagerstam [29] in a different context.

an AdS background. In comparison to standard higher spin theory one finds additional massive fields, whose numbers depend on the coupling. The construction of these theories in terms of free Yang-Mills theory provides evidence for their existence and finiteness, but the formulation is very primitive and on-shell, just like string theory. Hopefully, Vasiliev's [14] more geometric formulation may improve the understanding, perhaps also of large N gauge theory.

We have also discussed the non-trivial physics of these models, notably the existence of several massless spin 2 fields, and signs of non-perturbative bound states analogous to black holes.

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